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Comparison of standard Bayesian and Bayesian Expectation estimator to estimate parameters of the Kumaraswamy Distribution

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Abstract:
In this paper, the parameters of kumaraswamy distribution and the survival function were estimated using Bayesian methods, which is the standard Bayes method and the Bayesian expectation method with symmetrical loss functions called the squared loss function and asymmetrical loss functions called the general entropy loss function. The non-linear equations were evaluated using Lindley approximation using the simulation of Matlab program, the lowest value of the mean integral error squares was used to decide which method is the best.

فی هذا البحث تم تقدیر معلمات توزیع کوماراسوامی ومن ثم إیجاد دالۀ البقاء باستعمال الطرائق البیزیۀ وهی طریقۀ بیز القیاسیۀ وطریقۀ توقع بیز ضمن دوال خسارۀ متماثلۀ تدعی بدالۀ الخسارۀ التربیعیۀ وغیر متماثلۀ تدعی بدالۀ خسارۀ انتروبی عامۀ، تم تبسیط المعادلات الغیر خطیۀ باستعمال تقریب لیندلی ومن خلال المحاکاۀ فی برنامج الماتلاب تم التوصل الی اقل قیمۀ لمتوسط مربعات الخطأ التکاملی والذی یعتمد علیه لمعرفه أی الطرق هی الأفضل .

Introduction:

Bayesian methods are much better and more exact than the traditional methods. Researchers in recent times interest in using the Bayesian methods to estimating the survival function, which represents the duration of an organism's survival until death, and since the kumaraswamy distribution is one of the distributions that are interested in studying data The parameters of life were estimated using the standard Bayes method and the Bayesian exectation method under the quadratic loss and general entropy functions, to estimate the survival function for the kumaraswamy distribution.

1 - Kumaraswamy Distribution(KD) [2]

A continuous probability distribution consisting of two parameters that was proposed by Bundi kumaraswamy, one of the great Indian scientists and engineers. A random variable x has a KSD distribution if its p. d. f, as in equation 1

$$f(x;\propto,\beta) - \begin{cases} \propto \beta x^{\propto-1}(1-x^{\propto})^{\beta-1}\,; 0<x<1\,;\ \alpha,\beta>0 \\ 0\ otherwise\ 2-2 \end{cases} \quad (1)$$

It is similar to the betta distribution, but unlike the betta distribution, it contains a closed form of the cumulative distribution function, which facilitates handling and makes it appropriate for intensive computing activities such as modeling and simulation. It is applicabile to many natural phenomena that have a bounded response (lower and upper limits) such as the height of people, temperature, etc.

2- Loss function : [5][8]

The loss function is used to determine the error between the output of the algorithm and the specific target value, and it has an important role in Bayesian estimation, that is because Bayesian estimations differ according to the different types of loss functions.

2-1 Squared error Loss function : It is a symmetric loss function that can be expressed as in equation 2

$$L(\sigma^{\wedge},\sigma) = (\sigma^{\wedge}-\sigma)^2 \ 2$$

The Bayesian estimator for σ based on the loss function SE can be obtained as follows:

$$\hat{\sigma}_{SE} = E(\sigma|X) \ 3$$

2-2 General Entropy Loss function (GE) :

It is a modification of the linear exponential loss function (LINEX) proposed by (Varian, 1975) and (Zellner, 1986) and used by (Calabria and

Pulcini, 1994). It is classified as an asymmetric loss function that can be expressed mathematically as in equation 4

$$L(\sigma^{\wedge},\sigma)\alpha(\sigma^{\wedge}-\sigma)^{q}-q\ln(\sigma^{\wedge}|\sigma)-1\,,q\neq 0 \quad 4$$

We can get a Bayesian estimator with respect to the GE function mentioned earlier as follows:

$$\sigma^{\wedge}_{GE}=|E\sigma(\sigma^{-q})|^{-\frac{1}{q}} \quad 5$$

3. Standard Informative Bayesian Estimator: [1][٧][٣]

To find the standard Bayes estimator, which depends on the posterior distribution function, which includes the previous information of the parameter and the current sample observations, we use one of the Loss Functions, and it is considered one of the best ways to judge the performance of the estimated parameter

3-1 Standard Bayesian Estimator Under SE Loss function [4][1]

The standard Bayesian estimator SB for parameter ϴ can be defined as the Posterior mean of the random parameter ϴ. The SB method can be obtained for the parameters of the kumaraswamy distribution using the Prior Distribution function and the Squared Error Loss function. It was previously defined through the application of Lindley's approximate equations, as it is considered one of the best ways to simplify complex integrals, as well as because it gives accurate results.

$$\pi_1(\alpha)=\frac{\Gamma(a_1+b_1)}{\Gamma(a_1)\Gamma(b_1)}\alpha^{a_1-1}(1-\alpha)^{b_1-1}\,,0<\alpha<1 \quad 6$$

$$\pi_2(\beta)=\frac{\Gamma(a_2+b_2)}{\Gamma(a_2)\Gamma(b_2)}\beta^{a_2-1}(1-\beta)^{b_2-1}\,,0<\beta<1 \quad 7$$

Then we find the joint priority function, which represents the product of the initial probability density functions that were imposed above, as follows:

$$\pi_1(\alpha)\pi_2(\beta)=\frac{\Gamma(a_1+b_1)}{\Gamma(a_1)\Gamma(b_1)}\alpha^{a_1-1}(1-\alpha)^{b_1-1}\,\frac{\Gamma(a_2+b_2)}{\Gamma(a_2)\Gamma(b_2)}\beta^{a_2-1}(1-\beta)^{b_2-1} \quad 8$$

Let

$$A=\frac{\Gamma(a_1+b_1)}{\Gamma(a_1)\Gamma(b_1)}\,\frac{\Gamma(a_2+b_2)}{\Gamma(a_2)\Gamma(b_2)}$$

$$\pi_1(\alpha)\pi_2(\beta)=A\alpha^{a_1-1}(1-\alpha)^{b_1-1}\,\beta^{a_2-1}(1-\beta)^{b_2-1} \quad 9$$

Note that the possible function for the observations X1,X2,….,Xn is written as follows:

$$L = \prod_{i=1}^{n} f_{KSD}(x; \alpha, \beta)$$
$$= \prod_{i=1}^{n} \alpha\beta x^{\alpha-1}(1 - x^{\alpha})^{\beta-1}$$
$$= (\alpha\beta)^n x^{n(\alpha-1)}(1 - x^{\alpha})^{n(\beta-1)}$$

So that the joint distribution function:

$$\pi(\alpha,\beta)L = A\alpha^{a_1-1}(1-\alpha)^{b_1-1}\,\beta^{a_2-1}(1-\beta)^{b_2-1}\,(\alpha\beta)^n x^{n(\alpha-1)}(1 - x^{\alpha})^{n(\beta-1)}$$

$$f(x_1, x_1, \ldots, x_n) = \iint_{\forall\alpha\forall\beta} A\,\alpha^{a_1-1}(1-\alpha)^{b_1-1}\,\beta^{a_2-1}(1-\beta)^{b_2-1}\,(\alpha\beta)^n x^{n(\alpha-1)}(1 - x^{\alpha})^{n(\beta-1)}$$

The subsequent distributions of the parameters α,β will be as follows

$$h\,(\theta, \alpha, \beta|xi) = \frac{\prod_{i=1}^{n} f(xi, \alpha, \beta)\,\pi_1(\alpha)\,\pi_2(\beta))}{\int_\alpha\int_\beta \prod_{i=1}^{n} f(xi, \alpha\,\beta)\,\pi_1(m)\,\pi_2(\alpha)\,\pi_3(\beta)\,d\beta\, d\alpha} \quad 10$$

$$h\,(m, \alpha, \beta|\vec{x}) = \frac{A\,\alpha^{a_1-1}(1-\alpha)^{b_1-1}\,\beta^{a_2-1}(1-\beta)^{b_2-1}\,(\alpha\beta)^n x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}}{\int_\alpha\int_\beta A\,\alpha^{a_1-1}(1-\alpha)^{b_1-1}\,\beta^{a_2-1}(1-\beta)^{b_2-1}\,(\alpha\beta)^n x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}\,d\beta\,d\alpha}$$

$$h1\,(\alpha|\beta, \vec{x}) = \frac{\alpha^{a_1-1}(1-\alpha)^{b_1-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}}{\int_\alpha \alpha^{a_1-1}(1-\alpha)^{b_1-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}\,d\alpha} \quad 11$$

$$h2\,(\beta|\alpha, \vec{x}) = \frac{\beta^{a_2-1}(1-\beta)^{b_2-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}}{\int_\beta \beta^{a_2-1}(1-\beta)^{b_2-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}\,d\beta} \quad 12$$

The Bayes estimator under the squared loss function, which makes the risk function as minimum as possible, which represents the expectation of the loss function after finding the first derivative with respect to the parameter to be estimated and equalizing it to zero, we get:

$$Risk = E(d(\delta) - \hat{d}(\delta))^2)$$
$$= \int_{\forall\delta} (d(\delta) - \hat{d}(\delta))^2 h\,(\theta, \alpha, \beta|\vec{x})\,d\delta$$
$$= \int_{\forall\delta} (d(\delta)^2 - 2d(\delta)\hat{d}(\delta) + \hat{d}(\delta)^2)\,h\,(\theta, \alpha, \beta|\vec{x})\,d\delta$$
$$= \hat{d}(\delta)^2 - 2\hat{d}(\delta)E\big(d(\delta)|\underline{x}\big) + E\big(d(\delta)^2|\underline{x}\big) \quad 13$$

Differentiating equation (13) with respect to d̂(δ) and equating the derivative to zero, we get:

$$\hat{d}(\delta)_{SEL} = E_{\delta}(\delta|\underline{x}) \quad 14$$

Therefore, the standard Bayes estimator for the parameters of the Kumaraswamy distribution is as follows:

$$\hat{\alpha}_{SBSEL} = \frac{\partial}{\partial\hat{\alpha}}\left[\int_m \left((\alpha \quad \hat{\alpha})^2\right) h_1\,(\alpha|\,\beta,\vec{x})\,d\alpha\right] = 0$$

$$= \frac{\partial}{\partial\hat{\alpha}}\left[\int_\alpha (\alpha - \hat{\alpha})^2 \frac{\alpha^{a_1-1}(1-\alpha)^{b_1-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^\alpha)^{n(\beta-1)}}{\int_\alpha \alpha^{a_1 \quad 1}(1-\alpha)^{b_1 \quad 1}(\alpha\beta)^n x^{n(\alpha \quad 1)}(1-x^\alpha)^{n(\beta \quad 1)}\,d\alpha}\,d\alpha\right] = 0 \quad 15$$

$$\hat{\beta}_{SBSEL} = \frac{\partial}{\partial\hat{\beta}}\left[\int_\beta \left(\beta-\hat{\beta}\right)^2 h_2\,(\beta|\,\alpha,\vec{x})\,d\beta\right] = 0$$

$$= \frac{\partial}{\partial\hat{\beta}}\left[\int_\beta \left(\beta-\hat{\beta}\right)^2 \frac{\beta^{a_2-1}(1-\beta)^{b_2-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^\alpha)^{n(\beta-1)}}{\int_\beta \beta^{a_2-1}(1-\beta)^{b_2-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^\alpha)^{n(\beta-1)}\,d\beta}\,d\beta\right] = 0 \quad 16$$

Equations 15 and 16 are non-linear equations that cannot be solved by ordinary methods, so we will resort to the Lindley approximation method.

3-2 Standard Informative Bayesian Estimator under General Entropy Loss [1]

The standard Bayes estimator for the parameters of the Kumaraswamy distribution under general entropy loss function is given by:

$$\hat{\alpha}_{SDEL} = \frac{\partial}{\partial\hat{\alpha}}\left[\int_\alpha \left(\left(\frac{\hat{\alpha}}{\alpha}\right)^q - q\log\frac{\hat{\alpha}}{\alpha} - 1\right)^{\frac{-1}{q}} h_1\,(\alpha|\,m,\beta,\vec{x})\,d\alpha\right] = 0$$

$$= \frac{\partial}{\partial\hat{\alpha}}\left[\int_\alpha \left(\left(\frac{\hat{\alpha}}{\alpha}\right)^q - q\log\frac{\hat{\alpha}}{\alpha} - 1\right)^{\frac{-1}{q}} \frac{\alpha^{a_1-1}(1-\alpha)^{b_1-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^\alpha)^{n(\beta-1)}}{\int_\alpha \alpha^{a_1-1}(1-\alpha)^{b_1-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^\alpha)^{n(\beta-1)}\,d\alpha}\,d\alpha\right]$$

$$\hat{\beta}_{SBEL} = \frac{\partial}{\partial\hat{\beta}}\left[\int_\beta \left(\left(\frac{\hat{\beta}}{\beta}\right)^q - q\log\frac{\hat{\beta}}{\beta} - 1\right)^{\frac{-1}{q}} h_2\,(\beta|\,m,\alpha,\vec{x})\,d\beta\right] = 0$$

$$-\frac{\partial}{\partial\hat{\beta}}[\int_{\beta}\left(\left(\frac{\hat{\beta}}{\beta}\right)^{q}-q\log\frac{\hat{\beta}}{\beta}-1\right)^{\frac{-1}{q}}\frac{\beta^{a_2-1}(1-\beta)^{b_2-1}(\alpha\beta)^{n}x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}}{\int_{\beta}\beta^{a_2-1}(1-\beta)^{b_2-1}(\alpha\beta)^{n}x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}d\beta}d\beta$$

Equations 17 and 18 are non-linear equations, so we will resort to the Lindley approximation method.

4- Expected Bayesian Estimator :[1]

Choosing an initial probability density function that includes parameters that are chosen in such a way that the initial probability density function is decreasing with respect to the parameter to be estimated. The probability density functions for the parameters are as follows:

$$\pi(\alpha)=\frac{1}{k_1}\quad 0<\alpha<k_1 \quad 19$$

$$\pi(\beta)=\frac{1}{k_2}\quad 0<\beta<k_2 \quad 20$$

$$\pi^{*}(\alpha,\beta)\propto\frac{1}{k_1k_2}\quad 21$$

4-1Bayesian Expectation Estimator under Squared Loss Function (EBSEL)

According to the initial probability density function in equation (15) (16) and using the Bayesian prediction formula in equation (10), we get Bayes prediction estimations for the parameters of the kuomaraswamy distribution

$$\hat{\alpha}_{EBSEL}=\int_0^{k_1}\hat{\alpha}_{SBSEL}\pi(\alpha)\,d\alpha$$

$$\alpha_{EBSEL}=\int_0^{k_1}\frac{1}{k_1}\left(\frac{\partial}{\partial\hat{\alpha}}\left[\int_{\alpha}(\alpha-\hat{\alpha})^2\frac{\alpha^{a_1-1}(1-\alpha)^{b_1-1}(\alpha\beta)^{n}x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}}{\int_{\alpha}\alpha^{a_1-1}(1-\alpha)^{b_1-1}(\alpha\beta)^{n}x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}d\alpha}d\alpha\right]\right)d\alpha \quad 22$$

$$\hat{\beta}_{EBSEL}=\int_0^{k_2}\hat{\beta}_{SBSEL}\pi(\beta)d\beta$$

$$\beta_{EBSEL}=\int_0^{k_2}\frac{1}{k_2}\left(\frac{\partial}{\partial\hat{\beta}}[\int_{\alpha}(\beta-\hat{\beta})^2\frac{\beta^{a_2-1}(1-\beta)^{b_2-1}(\alpha\beta)^{n}x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}}{\int_{\beta}\beta^{a_2-1}(1-\beta)^{b_2-1}(\alpha\beta)^{n}x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}d\beta}d\beta]\right)d\beta \quad 23$$

We note that equations (23) and (22) are non-linear equations, and they cannot be solved by ordinary analytical methods, but their solution requires the use of numerical analysis methods, so Lindely Approximation will be used.

4-2 Bayesian prediction estimator given a general entropy loss function
The Bayesian prediction estimates for the kumaraswamy distribution under the general entropy loss function are as follows:

$$\hat{\alpha}_{EBSEL} - \int_0^{k_2} \hat{\alpha}_{SBEL}\pi(\alpha)d\alpha$$

$$\hat{\alpha}_{EBSEL} - \int_0^{k_2} \frac{1}{k_1}\left(\frac{\partial}{\partial\hat{\alpha}}\left[\int_\alpha \left(\left(\frac{\hat{\alpha}}{\alpha}\right)^q - q\log\frac{\hat{\alpha}}{\alpha} - 1\right)^{\frac{-1}{q}} \frac{\alpha^{a_1-1}(1-\alpha)^{b_1-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}}{\int_\alpha \alpha^{a_1-1}(1\quad\alpha)^{b_1-1}(\alpha\beta)^n x^{n(\alpha-1)}(1\quad x^{\alpha})^{n(\beta-1)}d\alpha} d\alpha\right]\right) d\alpha \; 24$$

$$\hat{\beta}_{EBSEL} = \int_0^{k_3} \hat{\beta}_{SBEL}\pi(a_2)d\beta$$

$$\hat{\beta}_{EBSEL} = \int_0^{k_2} \frac{1}{k_2}\left(\int_\beta \left(\left(\frac{\hat{\beta}}{\beta}\right)^q - q\log\frac{\hat{\beta}}{\beta} - 1\right) \frac{\beta^{a_2\ 1}(1-\beta)^{b_2\ 1}(\alpha\beta)^n x^{n(\alpha\ 1)}(1-x^{\alpha})^{n(\beta\ 1)}}{\int_\beta \beta^{a_2-1}(1-\beta)^{b_2-1}(\alpha\beta)^n x^{n(\alpha-1)}(1-x^{\alpha})^{n(\beta-1)}d\beta} d\beta\right) d\beta \; 25$$

We note that equations (24) and (25) are non-linear equations, and they cannot be solved by ordinary analytical methods, but their solution requires the use of numerical analysis methods, so Lindely Approximation will be used.

5-Simulations by Monte-Carlo method :[۶]

In order to compare the efficiency of the informative standard Bayes method and the Bayesian Expected method to obtain good estimates of the parameters of the Kumarasoa distribution, the simulation method was used by Monte Carlo, noting that the experiment was repeated (1000) using the MATLAB program, and the following is a detailed presentation of the experiments

Table (1-10) the real and estimated values of the survival function according to the estimation methods and the value of the mean integral error squares (IMSE) for each method at the assumed sample sizes for the first model:

Model 1	α=0.3 β =0.2					
n	t	Real(S(t))	$\hat{S}(t)_{SBSEL}$	$\hat{S}(t)_{SBEL}$	$\hat{S}(t)_{EBSEL}$	$\hat{S}(t)_{EBEL}$
٣0	0.1	0.745321	0.705818	0.685364	0.672323	0.783435
	0.2	0.541337	0.489691	0.462171	0.444628	0.582476
	0.3	0.386184	0.335791	0.308435	0.29106	0.418369
	0.4	0.271971	0.228417	0.204488	0.189371	0.293226
	0.5	0.189738	0.154526	0.135038	0.122799	0.201792
	0.6	0.131447	0.104156	0.0889861	0.079514	0.136914
	0.7	0.0905933	0.0700396	0.05859	0.0514792	0.0918504
	0.8	0.0621986	0.0470331	0.0385792	0.0333534	0.0610521
	0.9	0.0425848	0.0315626	0.0254202	0.0216382	0.0402701
	1	0.0290984	0.0211778	0.0167681	0.0140615	0.0263903
MSE			0.00177	0.07983	0.00278	0.00128
Best			$\hat{S}(t)_{EBEL}$			
60	0.1	0.745321	0.810676	0.772506	0.737042	0.720735
	0.2	0.541337	0.607982	0.550718	0.51052	0.492406
	0.3	0.386184	0.435364	0.375177	0.341061	0.325356
	0.4	0.271971	0.302448	0.248489	0.222781	0.210245
	0.5	0.189738	0.205719	0.161583	0.143433	0.133787
	0.6	0.131447	0.137795	0.103789	0.0914921	0.0842188
	0.7	0.0905933	0.0912473	0.0661183	0.0580213	0.0526142
	0.8	0.0621986	0.0599	0.0418912	0.0366705	0.0326978
	0.9	0.0425848	0.0390596	0.0264498	0.0231384	0.0202501
	1	0.0290984	0.810676	0.0166666	0.0145945	0.0125149
MSE			0.00034	0.01002	0.000677	0.000433
Best			$\hat{S}(t)_{SBSEL}$			
		0.745321	0.740216	0.732319	0.547184	0.6473426
90	0.2	0.541337	0.590415	0.531399	0.497684	0.474269
	0.3	0.386184	0.424481	0.370186	0.338997	0.316986
	0.4	0.271971	0.297625	0.253276	0.227641	0.209245
	0.5	0.189738	0.204982	0.171031	0.151274	0.136871
	0.6	0.131447	0.139328	0.114361	0.0997357	0.0889226
	0.7	0.0905933	0.0937634	0.0758909	0.0653558	0.0574767
	0.8	0.0621986	0.06262	0.0500633	0.0426208	0.0370078
	0.9	0.0425848	0.0415743	0.0328691	0.027687	0.023759
	1	0.0290984	0.0274755	0.0214974	0.0179289	0.01522
MSE			0.000228	0.00203	0.000163	0.000122
Best			$\hat{S}(t)_{EBEL}$			
150	0.1	0.745321	0.796275	0.748659	0.748659	0.704961
	0.2	0.541337	0.591852	0.531625	0.531625	0.476691
	0.3	0.386184	0.422248	0.3655	0.3655	0.314185
	0.4	0.271971	0.293313	0.246008	0.246008	0.203667
	0.5	0.189738	0.200029	0.163183	0.163183	0.130566
	0.6	0.131447	0.134619	0.107129	0.107129	0.0830737
	0.7	0.0905933	0.0897163	0.0698053	0.0698053	0.0525855
	0.8	0.0621986	0.0593519	0.0452369	0.0452369	0.0331721
	0.9	0.0425848	0.0390438	0.0291979	0.0291979	0.0208792
	1	0.0290984	0.0255732	0.0187903	0.0187903	0.0131243
MSE			0.000115	0.000191	0.000146	0.000121
Best			$\hat{S}(t)_{EBEL}$			

Through the use of the statistical criterion, the mean squares of the integral error, to compare the preference of the methods used to estimate the survival function for different sample sizes, the results were as follows:
At a sample size of (30) (90) (150) was the Bayesian prediction method under a general entropy loss function 〖S ̂(t)〗 _EBEL is the best in estimating the survival function because it has the lowest mean squared integral error (IMSE)
At a sample size of (60), the standard Bayes method was informative with a squared loss function 〖S ̂(t)〗 _SBSEL It is the best in estimating the survival function because it recorded the lowest mean integral error squares (IMSE).
Table (2-10) the real and estimated values of the survival function according to the estimation methods and the value of the mean integral error squares (IMSE) for each method at the assumed sample sizes for the first modelat the size of (30), the Bayesian prediction method under the squared loss function was the best in estimating the survival function, because it recorded the least mean squared integral error (IMSE).

Model	α=0.3 β =0.1					
n	t_i	Real(S(t))	$\ddot{S}(t)_{SBSEL}$	$\ddot{S}(t)_{SBEL}$	$\ddot{S}(t)_{EBSEL}$	$\ddot{S}(t)_{EBEL}$
30	0.1	0.713455	0.67946	0.66788	0.653714	0.653714
	0.2	0.481911	0.448694	0.434819	0.417895	0.417895
	0.3	0.314068	0.29082	0.27844	0.263435	0.263435
	0.4	0.199579	0.186136	0.176363	0.164619	0.164619
	0.5	0.124468	0.118119	0.110905	0.102325	0.102325
	0.6	0.0765064	0.0745236	0.0694184	0.0634144	0.0634144
	0.7	0.0464863	0.0468388	0.0433266	0.0392449	0.0392449
	0.8	0.0279811	0.0293678	0.0269988	0.0242795	0.0242795
	0.9	0.0167113	0.0183882	0.0168129	0.015027	0.015027
	1	0.00991501	0.0115064	0.0104695	0.00930874	0.00930874
MSE			0.0685807	0.032741	0.0100305	0. 0119667
Best			$\ddot{S}(t)_{EBSEL}$			
60	0.1	0.713455	0.74405	0.737934	0.722026	0.693336
	0.2	0.481911	0.505135	0.48941	0.470167	0.445661
	0.3	0.314068	0.325618	0.307061	0.290122	0.274147
	0.4	0.199579	0.203145	0.186513	0.173512	0.164016
	0.5	0.124468	0.123982	0.111049	0.10181	0.0963347
	0.6	0.0765064	0.0745148	0.0652923	0.0590392	0.0558819
	0.7	0.0464863	0.0442974	0.0380893	0.033995	0.0321456
	0.8	0.0279811	0.0261283	0.0221164	0.0194975	0.0183908
	0.9	0.0167113	0.0153257	0.0128099	0.0111626	0.0104869
	1	0.00991501	0.00895457	0.00741239	0.00638897	0.00596996

MSE			0.00202771	0.00483703	0.00619474	0.00729251
Best			$\ddot{S}(t)_{SBEL}$			
90	0.1	0.713455	0.722651	0.710408	0.693197	0.670012
	0.2	0.481911	0.489803	0.476193	0.456865	0.430728
	0.3	0.314068	0.31927	0.308079	0.291817	0.269621
	0.4	0.199579	0.202806	0.1947	0.182537	0.165738
	0.5	0.124468	0.12653	0.121052	0.112518	0.100582
	0.6	0.0765064	0.0779268	0.0743756	0.0686195	0.0604774
	0.7	0.0464863	0.047539	0.0452947	0.041514	0.0361167
	0.8	0.0279811	0.028797	0.0273985	0.0249614	0.0214605
	0.9	0.0167113	0.0173526	0.0164863	0.0149367	0.0127047
	1	0.00991501	0.0104159	0.00987896	0.00890379	0.00750094
MSE			0.000166232	0.000269083	0.000202594	0.00025332
Best			$\ddot{S}(t)_{SBSEL}$			
150	0.1	0.713455	0.728549	0.714662	0.69783	0.677028
	0.2	0.481911	0.490124	0.47417	0.454895	0.431139
	0.3	0.314068	0.315306	0.301675	0.285269	0.265129
	0.4	0.199579	0.197289	0.186993	0.174649	0.159568
	0.5	0.124468	0.121203	0.113937	0.10526	0.0947144
	0.6	0.0765064	0.0735294	0.0686161	0.0627722	0.0557079
	0.7	0.0464863	0.0442146	0.0409878	0.0371644	0.0325678
	0.8	0.0279811	0.02642	0.0243446	0.0218943	0.0189645
	0.9	0.0167113	0.0157163	0.0144017	0.0128552	0.0110159
	1	0.00991501	0.00931937	0.00849649	0.00753149	0.00638969
MSE			0.000120205	0.0000953118	0.000103675	0.000127982
Best			$\ddot{S}(t)_{SBEL}$			

It is clear from Table (2-10) and Figures from (5) to (8) and when the default values of the parameters and through the use of the statistical standard mean squares of integral error to compare the preference of the methods used to estimate the survival function for different sample sizes, the results were as follows:

At a sample size of (60) (150), the standard information Bayes method under a general entropy loss function was the best in estimating the survival function, because it recorded the least mean squares integral error IMSE

At a sample size of (90), the standard Bayes method under the squared loss function was the best in estimating the survival function because it recorded the lowest mean squared integral error IMSE

Table (3-10) the real and estimated values of the survival function according to the estimation methods and the value of the mean integral error squares (IMSE) for each method at the assumed sample sizes for the first model

It is clear from Table (3-10) and Figures from (9) to (12) and when the default values of the parameters and through the use of the statistical standard mean integral error squares to compare the preference of the methods used to estimate the survival function for different sample sizes, the results were as follows:

∗ At a sample size of (30), the standard informational Bayes method under a general entropy loss function was the best in estimating the survival function, because it recorded the least mean squares integral error IMSE

∗ At a sample size of (60), the standard informational Bayes method with a squared loss function was the best in estimating the survival function, because it recorded the lowest mean squared integral error IMSE

∗ At a sample size of (90) (150), the Bayesian prediction method under a general entropy loss function was the best in estimating the survival function, because it recorded the lowest mean squared integral error IMSE.

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Appendices
Table (1-10) the real and estimated values of the survival function according to the estimation methods and the value of the mean integral error squares (IMSE) for each method at the assumed sample sizes for the first model

Figure (1) The real survival function estimated according to the estimation methods at a sample size (n = 10)

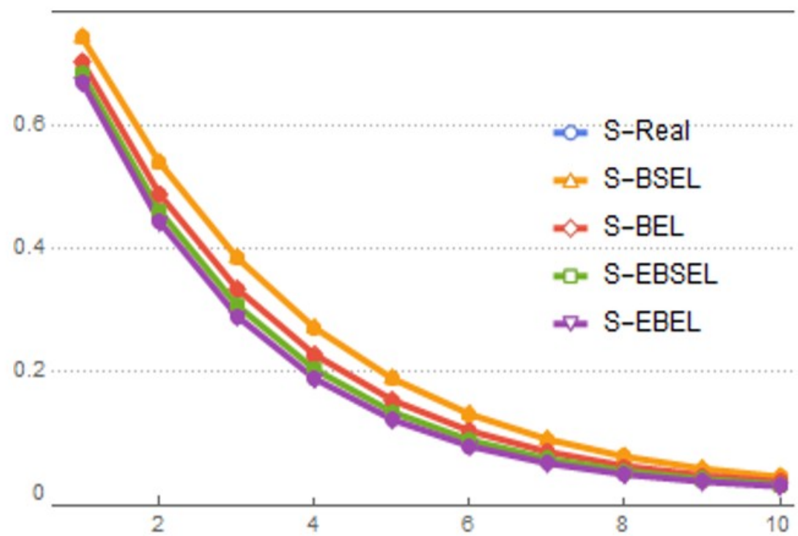


Figure (2) The real survival function estimated according to the estimation methods at a sample size (n = 60)

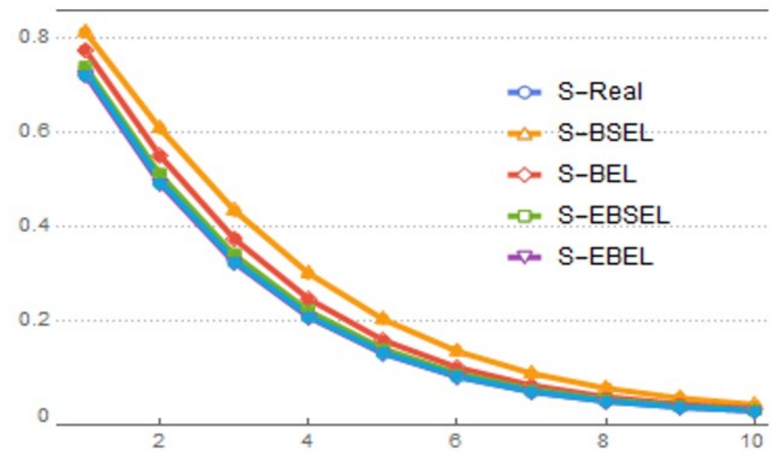


Figure (3) The true survival function estimated according to the estimation methods at a sample size (n = 90)

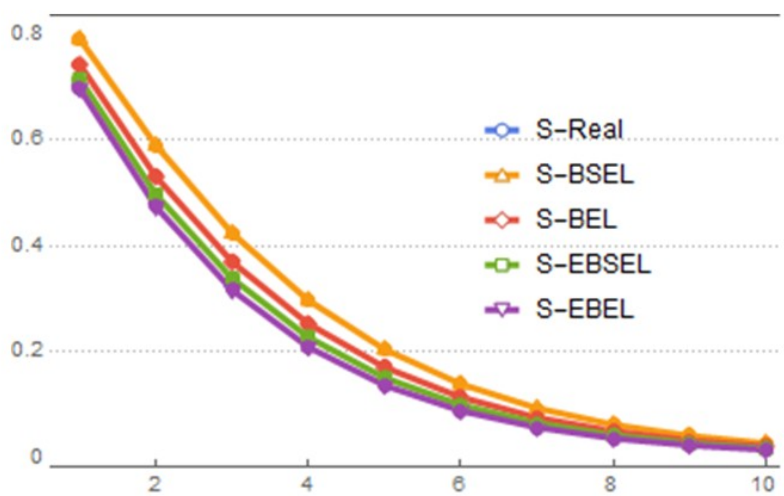


Figure (4) The true survival function estimated according to the estimation methods at a sample size (n = 150)

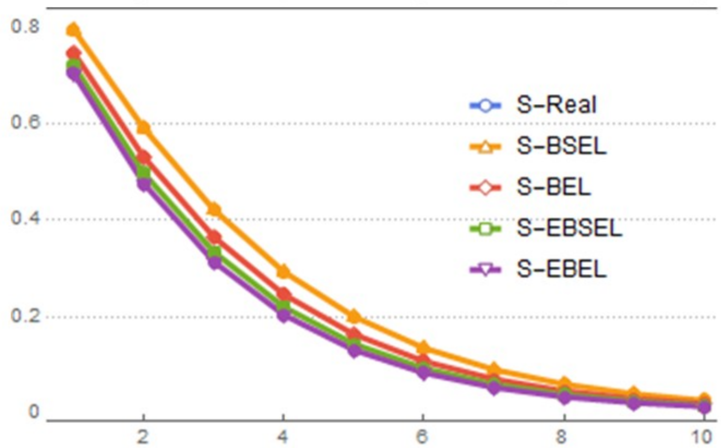


Figure (5) The real survival function estimated according to the estimation methods at a sample size (n = 30)

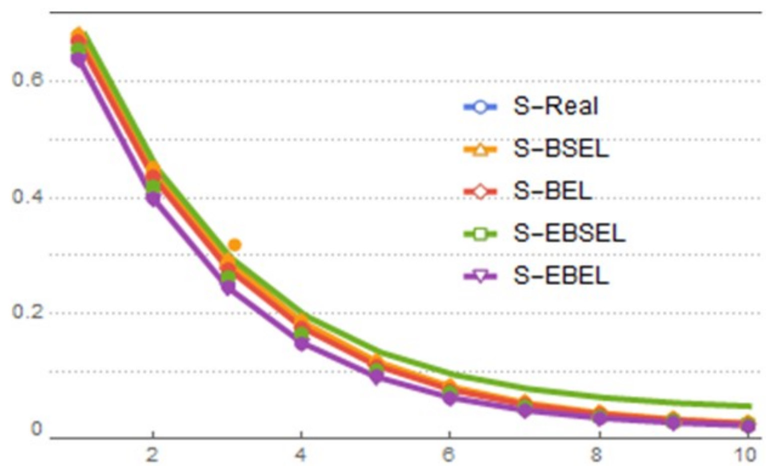


Figure (6) The real survival function estimated according to the estimation methods at a sample size (n = 60)

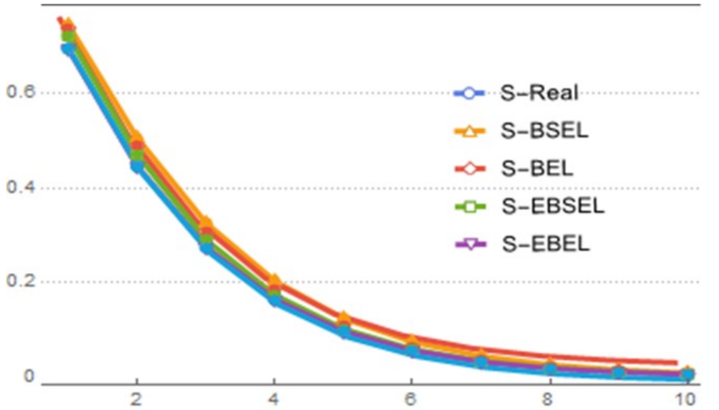


Figure (7) The real survival function estimated according to the estimation methods at a sample size (n = 90)

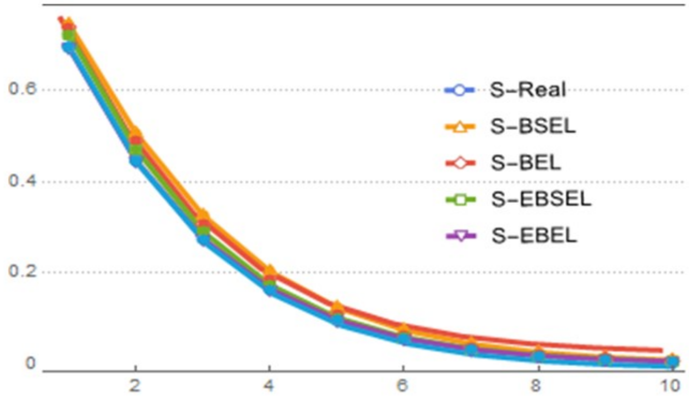
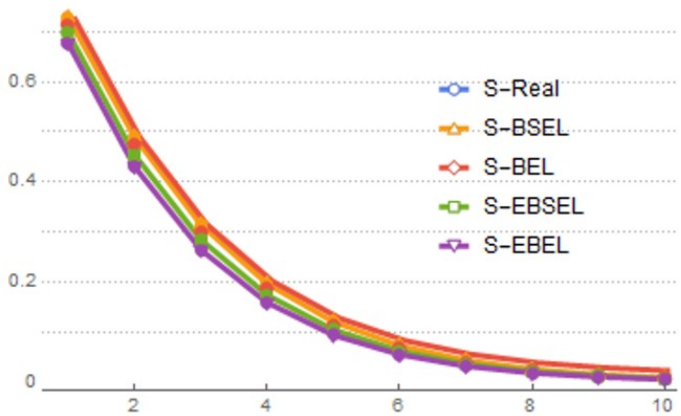


Figure (8) The real survival function estimated according to the estimation methods at a sample size (n = 150)



Model ۳	α=0.6 β =0.2					
n	t	Real(S(t))	$\hat{S}(t)_{SBSEL}$	$\hat{S}(t)_{SBEL}$	$\hat{S}(t)_{EBSEL}$	$\hat{S}(t)_{EBEL}$
30	0.1	0.856518	0.851982	0.780174	0.801307	0.83949
	0.2	0.732196	0.72332	0.604192	0.637703	0.697408
	0.3	0.624801	0.612311	0.465362	0.504829	0.574579
	0.4	0.532282	0.517105	0.357003	0.398016	0.470224
	0.5	0.452775	0.435848	0.273089	0.312821	0.382725
	0.6	0.384602	0.366774	0.208478	0.245275	0.310111
	0.7	0.326267	0.308246	0.158942	0.191969	0.25034
	0.8	0.276443	0.258788	0.121079	0.15005	0.201464
	0.9	0.233962	0.217087	0.0922019	0.117176	0.161711
	1	0.197798	0.181989	0.0702088	0.0914483	0.129522
MSE			0.00347954	0.00196119	0.00719529	0.00347954
Best			$\hat{S}(t)_{SBSEL}$			
60	0.1	0.856518	0.88034	0.819133	0.852384	0.890024
	0.2	0.732196	0.766338	0.652647	0.701824	0.76549
	0.3	0.624801	0.661104	0.510143	0.564325	0.642739
	0.4	0.532282	0.566129	0.393326	0.446118	0.530178
	0.5	0.452775	0.481841	0.300225	0.34826	0.431429
	0.6	0.384602	0.408005	0.227453	0.269287	0.347345
	0.7	0.326267	0.343987	0.171356	0.206701	0.277262
	0.8	0.276443	0.288944	0.128553	0.157761	0.219777
	0.9	0.233962	0.24194	0.096141	0.119876	0.173207
	1	0.197798	0.202031	0.0717361	0.0907754	0.135848
MSE			0.0009334171	0.00168218	0.00652952	0.00420721

Best			$\hat{S}(t)_{SBEL}$			
90	0.1	0.856518	0.864591	0.795715	0.825198	0.874234
	0.2	0.732196	0.744037	0.628072	0.671997	0.745394
	0.3	0.624801	0.637757	0.492648	0.541674	0.623976
	0.4	0.532282	0.544801	0.384514	0.433114	0.51508
	0.5	0.452775	0.464029	0.298926	0.34407	0.420553
	0.6	0.384602	0.394227	0.231642	0.271886	0.340373
	0.7	0.326267	0.334181	0.179027	0.213905	0.273517
	0.8	0.276443	0.28273	0.138059	0.167671	0.218499
	0.9	0.233962	0.238791	0.10627	0.131023	0.173689
	1	0.197798	0.201377	0.0816713	0.102114	0.137497
MSE			0.000633848	0.001559614	0.00429338	0.000233848
Best			$\hat{S}(t)_{EBEL}$			
150	0.1	0.856518	0.870841	0.803514	0.83125	0.879244
	0.2	0.732196	0.75182	0.63477	0.677021	0.750088
	0.3	0.624801	0.644569	0.495304	0.543286	0.626425
	0.4	0.532282	0.549502	0.382932	0.431141	0.514998
	0.5	0.452775	0.466281	0.293989	0.339231	0.41835
	0.6	0.384602	0.394137	0.224492	0.265136	0.336671
	0.7	0.326267	0.332081	0.17071	0.20613	0.268922
	0.8	0.276443	0.279036	0.129393	0.159578	0.213509
	0.9	0.233962	0.233926	0.0978298	0.123117	0.168674
	1	0.197798	0.195726	0.0738218	0.0947235	0.132706
MSE			0.000522843	0.000451596	0.000287817	0.000122843
Best			$\hat{S}(t)_{EBEL}$			

Figure (9) The real survival function estimated according to the estimation methods at a sample size (n = 30)

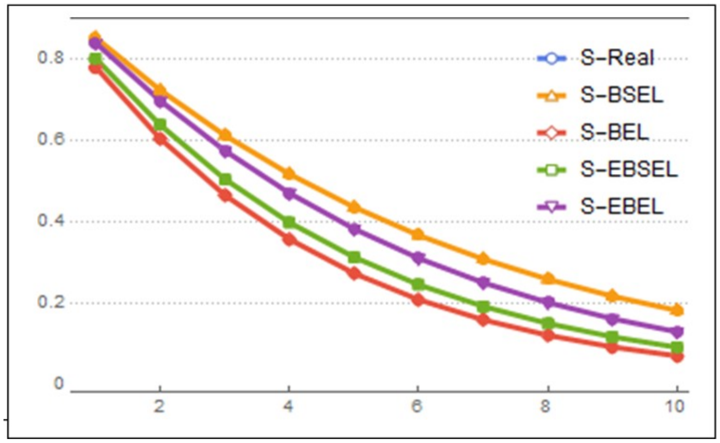


Figure (10) The real survival function estimated according to the estimation methods at a sample size (n = 60)

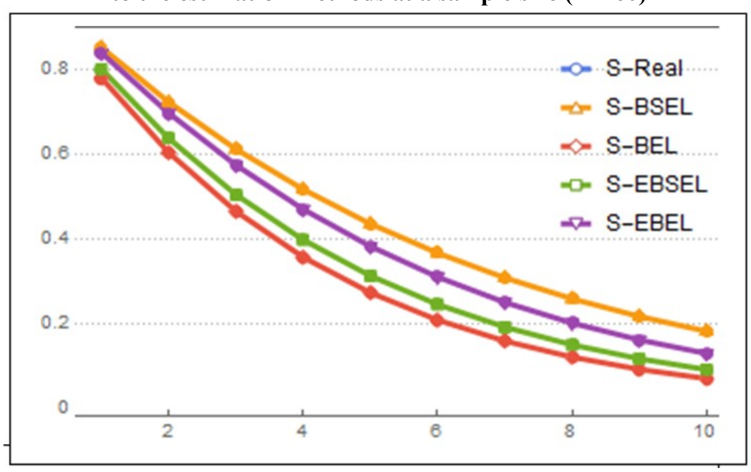


Figure (11) The real survival function estimated according to the estimation methods at a sample size (n = 90)

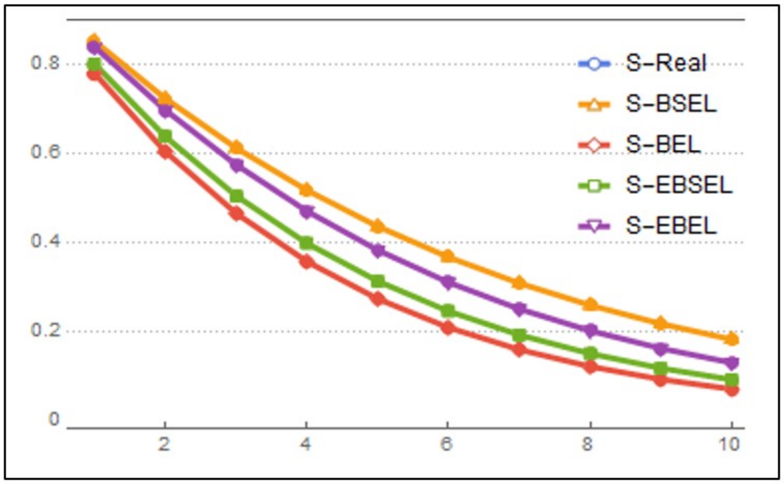


Figure (12) The real survival function estimated according to the estimation methods at a sample size (n = 150)